Let’s Stay Together: Towards Traffic Aware Virtual Machine Placement in Data Centers

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Outline

- Background and Scenario
- Problem Statement
- Homogeneous Case
- Heterogeneous Case
- Conclusion

Background

- Virtual machine (VM) placement
  - Tenants submit their resource requirements to the cloud system, and the cloud decides how to implement the resource allocation.
  - One of the primary task in virtualization-based cloud system.
- The cost is one of the major concerns for the cloud providers.
  - PM-cost
  - N-cost

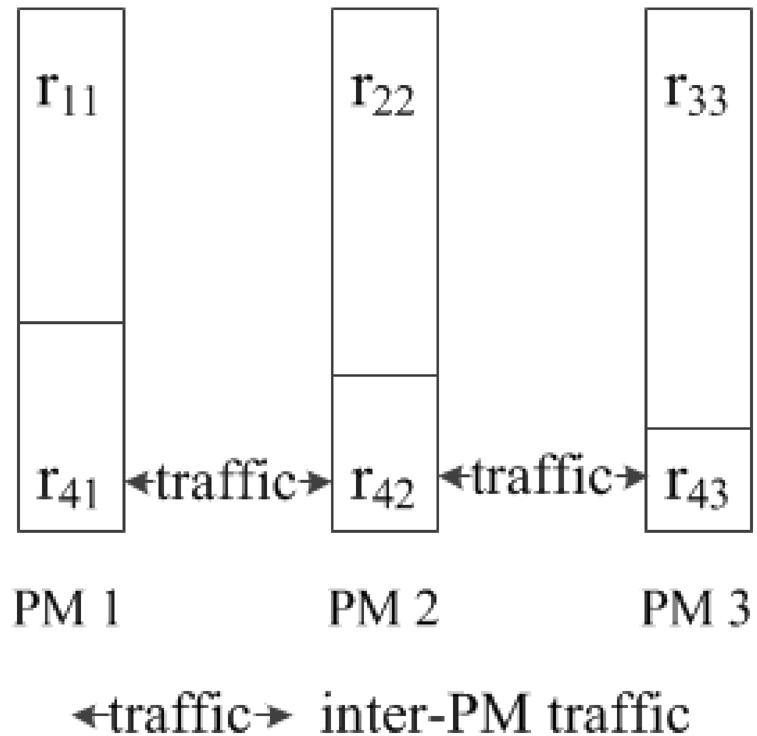
Scenario

- We use *slot* to represent one basic resource unit. (CPU/memory/disk)
- Tenants submit their resource requirements, in terms of the number of VMs (slots).
  - Each slot host one VM
- For one tenant, it could be one project group, and each VM can be assigned to one group member.
  - The VMs (group members) finish the task cooperatively.

Virtual Machine Placement

- Inter-PM traffic
  - Inter-VM traffic
- The objective
  - Minimize the total inter-PM traffic.

How to determine the communication cost?

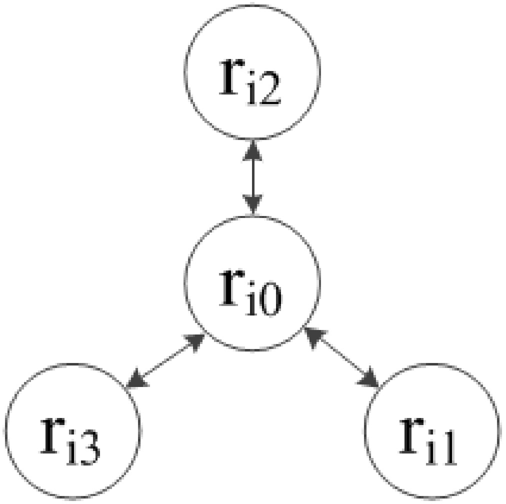


placement for 4 requests on 3 PMs: r_1, r_2, r_3, r_4.

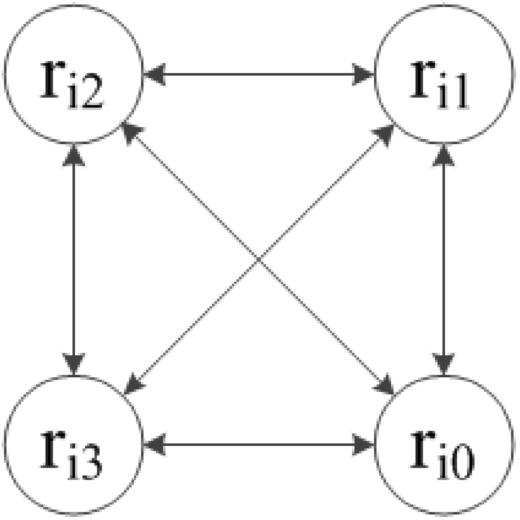
r_{ij}: the VMs placed on PM j of request r_i.

Communication Model

- Two communication models



Centralized
Model



Distributed
Model

r_{ij}: the VMs placed on PM j of request r_i.

Communication Cost

- The traffics between VMs are assumed to be aware in most related works.
- Here, we do NOT adopt this assumption.
- We focus on network cost, measured by the number of traffic links between VMs
  - One request may be placed on multiple servers

Problem Statement

- Given a set of requests $R = \{r_i | 0 \le i < n\}$, and a data center that consists of m uniform PMs with c slots for each. There may be traffic between VMs of the same tenant. Present a VM placement such that the overall network cost is minimized.
  - ϕ_i: the cost caused by request i
  - r_i: the requirement of request i
  - objective:

$$min \sum_{i=0}^{n-1} \phi_i$$

Cost Function

- Centralized Model Cost Function (CCF)
  - $\phi_i^{(1)} = K_i$
- Distributed Model Cost Function (DCF)
  - $\phi_i^{(2)} = K_i^2$
- Enhanced Distributed Model Cost Function (E-DCF)
  - $\phi_i^{(3)} = \frac{1}{2}\sum_{\kappa=1}^{K_i} r_i^{(\kappa)} \cdot (r - r_i^{(\kappa)})$

K_i: the number of fractions of request i.

Classification

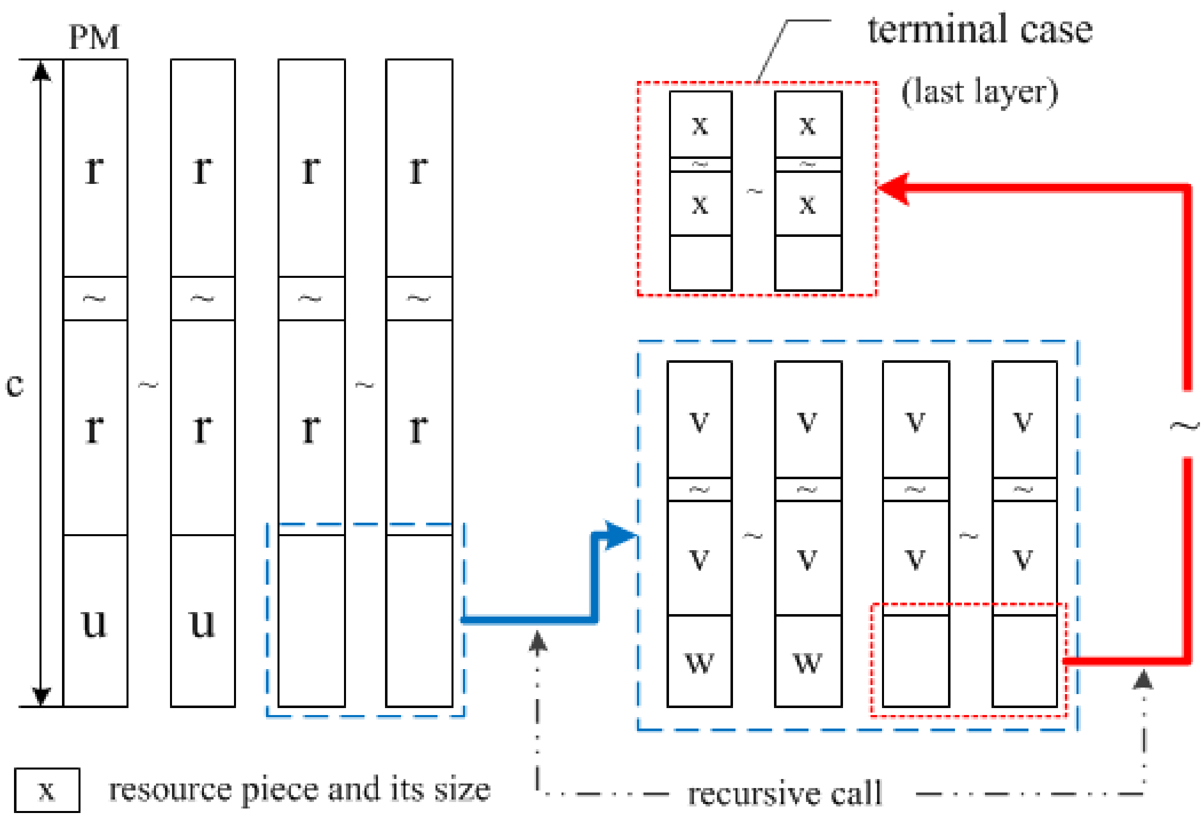
- Only N-cost is discussed, PM-cost is fixed as the minimal number of PMs that can host all of the required VMs.
- Homogeneous case
  – $r_i = r_j = r$;
- Heterogeneous case
  – otherwise.

Homogeneous Case

- CCF
  - Recursive algorithm
  - Optimal solution
- DCF
  - Algorithm based on the above recursive algorithm
  - Optimal solution
- E-DCF
  - Recursive algorithm
  - Optimal solution

Homogeneous Case - CCF

- Recursive algorithm



solution
structure

Homogeneous Case - CCF

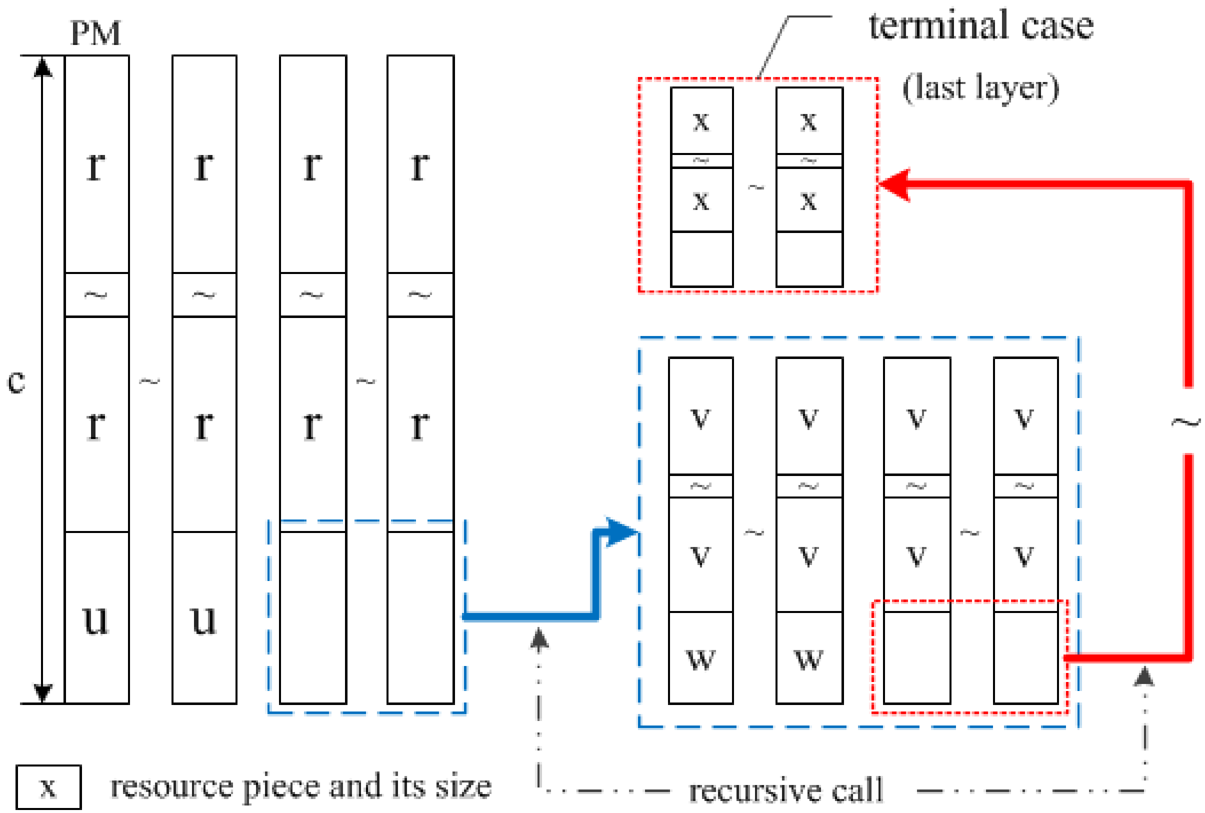
- Basic idea
  - Achieve the *perfect placement* as many as possible, then split the unplaced requests into *pieces*.
    - Layer
  - Perfect placement (Stay Together)
    - All of the required VMs are placed on the same PM.
    - For each layer, the perfect placement may be different, i.e. the number of required VMs varies.
  - Piece
    - TPC: *terminal-piece*
    - CPC: *continue-piece*

Homogeneous Case - CCF

- Piece
  - TPC, terminal piece
    - One piece is placed completely without split at some layer;
  - CPC, continue piece
    - Otherwise.

- There is exactly one TPC for each request.
- There is at most one CPC on each PM.

Solution Structure



solution
structure

$$c = \alpha \cdot r + u$$
$$r = \beta \cdot u + v$$
TPC: r
CPC: u

$$c \leftarrow u$$
$$r \leftarrow v$$
TPC: v
CPC: w

recursively

$$\boldsymbol{\phi}_i^{(1)} = \boldsymbol{K}_i$$

Homogeneous Case - CCF

- Swap operation
  - s_i: a set of pieces placed on PM i.
  - $swap(s_i, s_j)$
    - $s_i = s_j$
    - $s_i > s_j$
      - Split s_i into two parts, s_i^* and ${s_i}^\Delta$, such that $s_i^* = s_j$, then swap s_i^* and s_j.
      - It is easy to get s_i^* by splitting ONLY one piece into two parts.
    - $s_i < s_j$

Optimality

- Theorem
  - The recursive algorithm gives the optimal solution when $\forall i, r_i = r \leq c$, and $\phi_i = \phi_i^{(1)} = K_i$, i.e., the CCF cost function.
- Proof
  - Case Ω
    - For any PM, the sum of the sizes of the *fragments* is more than *r*.
  - Fragment

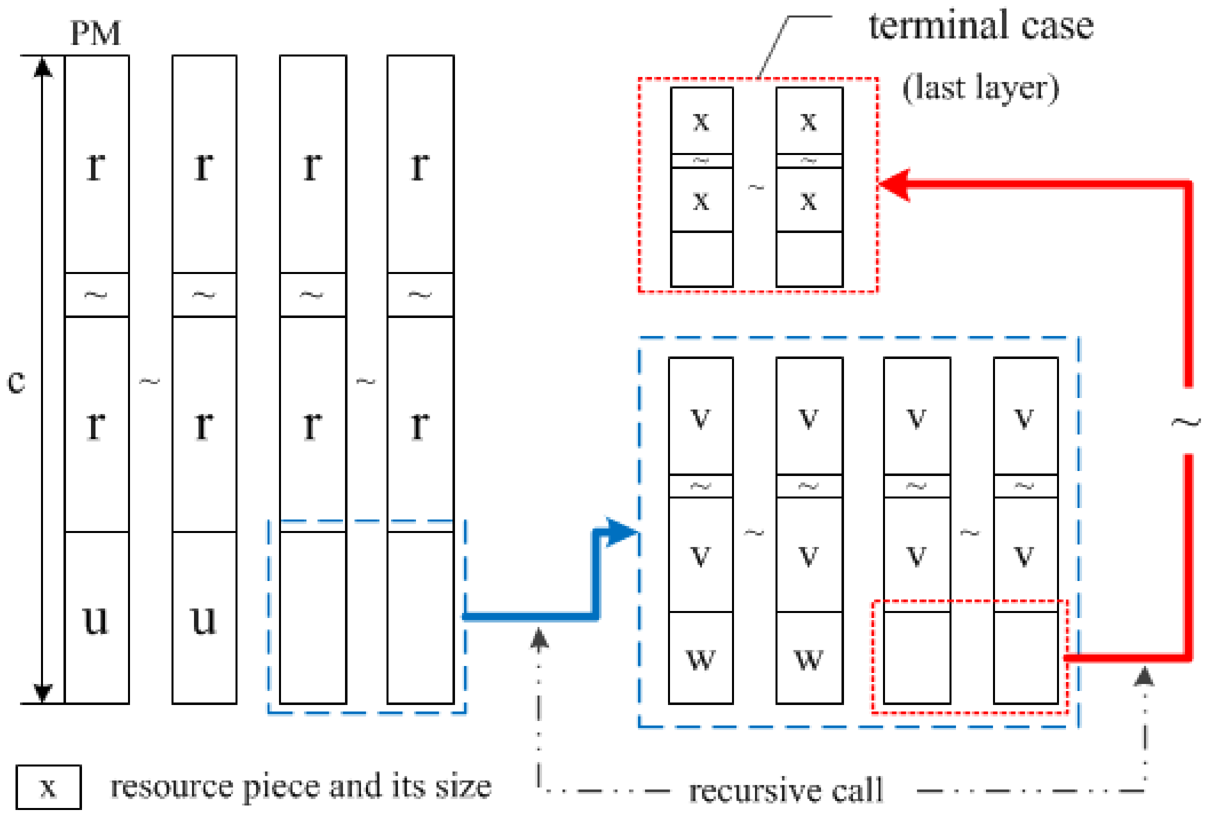
Proof

- There is no case Ω in our solution.
- In the optimal solution, we can remove all of the case Ω.
  - Let r_{ij} be one of the fragment, and s_j be the union of the other *fragments* of PM j.
  - There must be another fragment $r_{ij'}$ on PM j', and we have $s_j > r_{ij'}$ since $s_j + r_{ij} > r$.
  - Swap operation: $swap(s_j, r_{ij'})$.
  - The swap operation will not change the fact

Proof (cont.)

- Repeat the swap operation until there is only one piece for r_i, and the sum of the size of fragments on PM j can be reduced by r.
- There can be no case Ω in the optimal solution.
  - Reduce the optimal solution to our solution.
  - There are α perfect placement in the layer 0.
  - For the remaining pieces, we can do swap operation to gather the pieces of the same tenant as close as possible.

Solution Structure



solution structure

$$c = \alpha \cdot r + u$$
$$r = \beta \cdot u + v$$
TPC: r
CPC: u

$$c \leftarrow u$$
$$r \leftarrow v$$
TPC: v
CPC: w

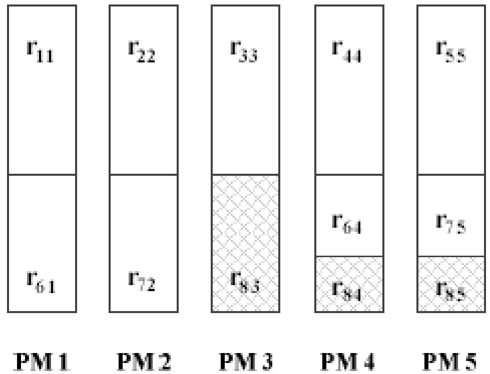
recursively

$$\boldsymbol{\phi}_i^{(1)} = \boldsymbol{K}_i$$

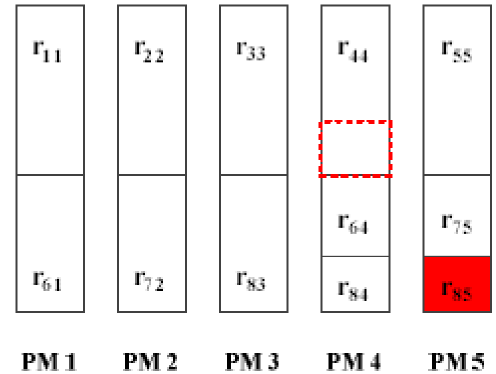
Homogeneous Case - DCF

- DCF: $\phi_i^{(2)} = K_i^2$
- CCF: the sum of the pieces is minimal.
- The basic idea
  - To minimize the objective function, we should achieve the K distribution like this: 1,1,…,1,2,…,2
    - Swap operation
  - For given number of items, to minimize the sum of the square of items, its sum should be minimized, and it achieves the minimal value when all the

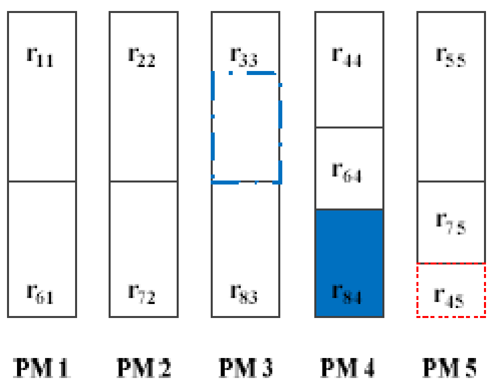
Homogeneous Case - Example



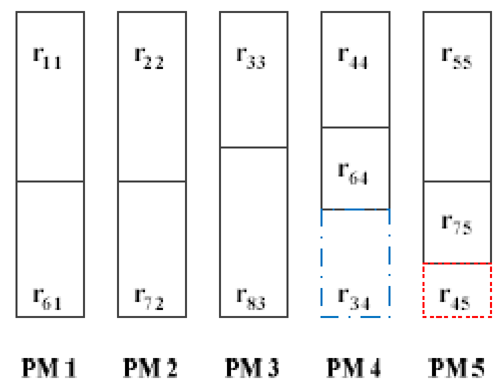
(a) Placement given by Algorithm 1. There are 2 layers, and $K_8 = 3$.



(b) The TPC of r_8 is located (red rectangle), and s_4 (red dashed rectangle) is selected.



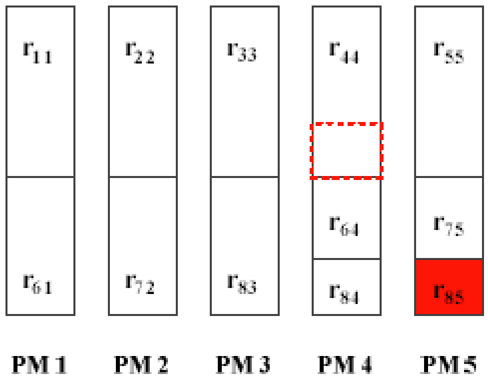
(c) Do $swap(r_{85}, s_4)$, then we have $K_3 = 2$,$K_8 = 2$. s_3 (blue dashed rectangle) is selected.



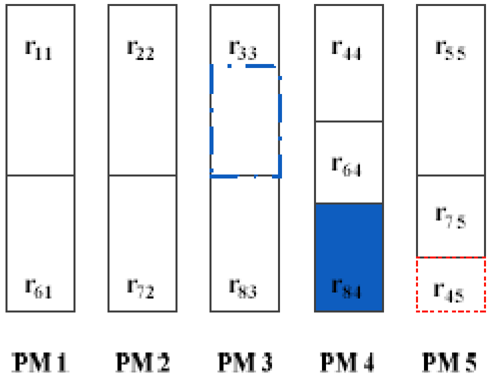
(d) Do $swap(r_{84}, s_3)$. We achieve the final optimal placement.

Optimality

- Feasibility of the swap operation.
  - There must be at least 1 perfectly placed request on the PMs that contains CPC of r_i.
  - The perfectly placed request will provide its part to be swapped out of the PM.



(b) The TPC of r_8 is located (red rectangle), and s_4 (red dashed rectangle) is selected.



(c) Do $swap(r_{85}, s_4)$, then we have $K_3 = 2, K_8 = 2$. s_3 (blue dashed rectangle) is selected.

Optimality (cont.)

- Let the swap operation start from the TPC of r_i, so it is unnecessary for the PM that contains TPC of r_i.
- Only one perfectly placed pieces on each PM is enough.
  - There is at most one CPC on each PM.
- In fact, we have $\alpha (\alpha > 1)$ perfectly placed pieces on each PM.

Optimality (cont.)

- After the swap operations for all requests that have more than 2 pieces, their piece number becomes to 1.
  - For the other request that participate the swap operation (the perfectly placed request), their piece number becomes to 2.
  - For the other, their piece number remains unchanged.
  - We achieve the optimal K distribution.

Homogeneous Case – E-DCF

- The same algorithm as the case CCF.
  - Recursive algorithm
  - $\phi_i^{(3)} = \frac{1}{2}\sum_{\kappa=1}^{K_i} r_i^{(\kappa)} \cdot (r - r_i^{(\kappa)})$
- We assume that $r_{iu}, r_{iv}, r_{ju}, r_{jv}$ are four pieces.
  - The four piece will not coexist in the optimal placement, because we can do $swap(r_{iu}, r_{jv})$ or $swap(r_{iv}, r_{ju})$.
  - If $r_{iu} \geq r_{iv}$ and $r_{iu} + r_{iv} > r_{ju}$, then r_{iu}, r_{iv}, r_{ju} will not coexist, since we can do $swap(r_{ju}, r_{iv})$.

Optimality

- From the two facts, we can construct the optimal solution from any give placement.
  – (1)Mark the pieces that have the size equal to r as red; otherwise, black.
  – (2)Select the piece with largest size among the black pieces. (Assume that r_{iu} is selected)
  – (3)Do $swap(r_{ju}, r_{iv})$, as shown above, until no r_{ju} or r_{iv} can be selected. Then mark the new r_{iu} red.

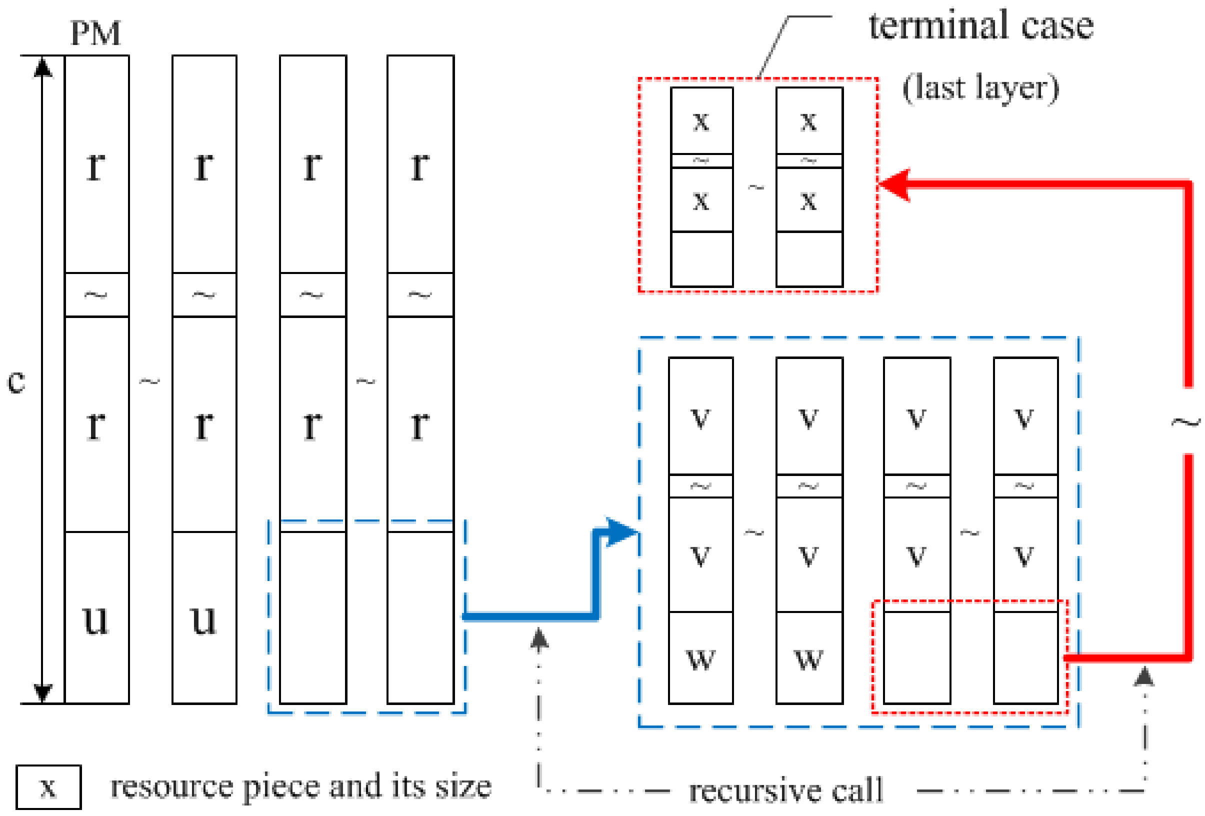
Optimality (cont.)

- The impact of the swap operation (step 3).
  - The piece r_{iu} will be larger.
- Feasibility of the swap operation.
  - r_{iu} has the largest size among the black pieces.
- When the swap operation will be terminated.
  - $K_i = 1$ ($r_{iu} = r$)
  - The other pieces on PM u are all marked as red.
    - If it still have black piece, the swap operation can continue.

Optimality (cont.)

- The red piece will not participate the swap operation.
  - The red piece has the size equal to r; (step 1)
  - There are no black pieces on the PM it located.
- From the construction process, there will be α perfect placement on each PM, and other requests will occupy as fewer PM as possible.
- The result matches the recursive solution.

Solution Structure



$$c = \alpha \cdot r + u$$
$$r = \beta \cdot u + v$$

TPC: r

CPC: u

$$c \leftarrow u$$
$$r \leftarrow v$$

TPC: v

CPC: w

recursively

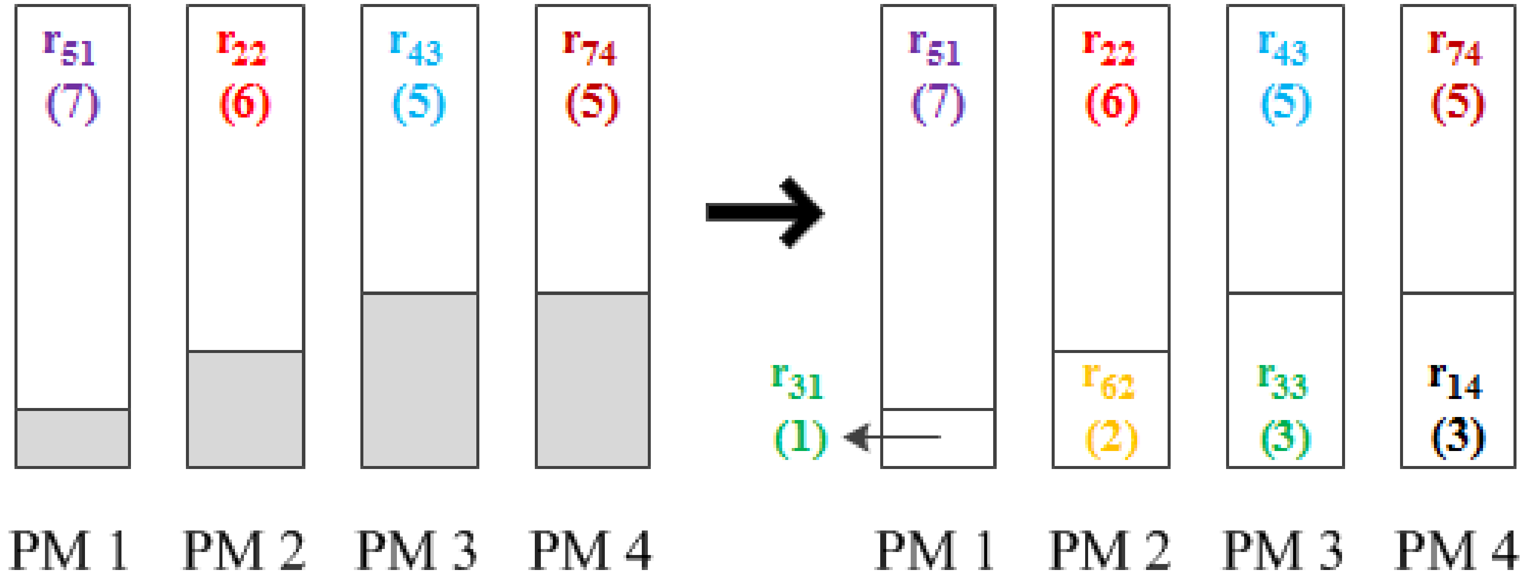
solution structure

Heterogeneous Case

- SBP: Sorting-based Placement
- Basic idea: place the requests with larger VM requirements first.
  - Sorting
    - According the number of VMs that tenants require
    - Ascending order
  - Place the first item of the sequence (r_0)
    - Case 1: perfect placement
    - Case 2: split r_0 into two pieces

An Example

- The inputs:
  - $r_1 = 3, r_2 = 6, r_3 = 4, r_4 = 5, r_5 = 7, r_6 = 2, r_7 = 5$
    - Different color
  - Sorting: 7， 6， 5， 5， 4， 3， 2



Greedy Algorithm

- Basic idea
  - The basic idea of GBP is that, for each request, place the required VMs on the current PM as much as possible; when the current PM is fully loaded, then place the part that exceeds the PM capacity to the next PM. Hence, there are at most 2 pieces for each request. In fact, the total number of pieces will not exceed $m + n$, since there are at most m requests that are split into two pieces.

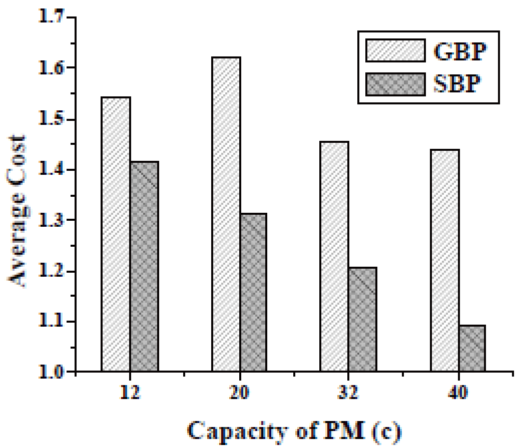
Approximation Ratio of GA

$$\sum_{i=0}^{n-1} \phi_i^{(1)} < m + n \leq 2 \cdot n \leq 2 \cdot OPT$$

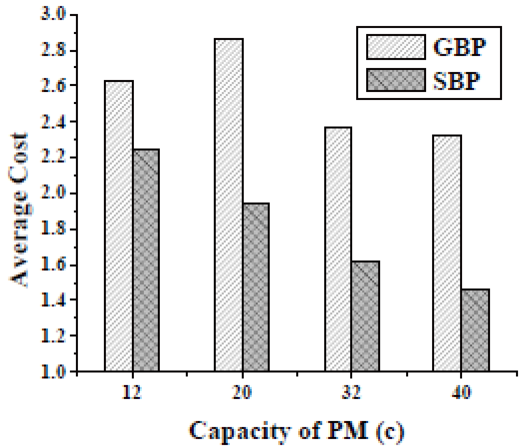
$$\sum_{i=0}^{n-1} \phi_i^{(2)} \leq 4 \cdot n \leq 4 \cdot OPT$$

$$\sum_{i=0}^{n-1} \phi_i^{(3)} \leq \sum_{i=0}^{n-1} \frac{r_i^2}{4} \leq \frac{c^2}{4} \cdot n \leq \frac{c^2}{4} \cdot OPT$$

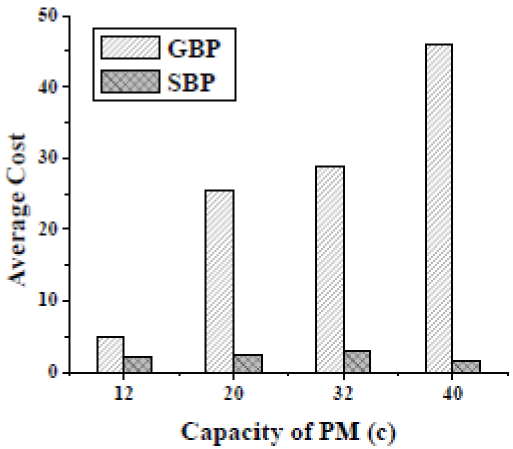
Comparison



(a) CCF

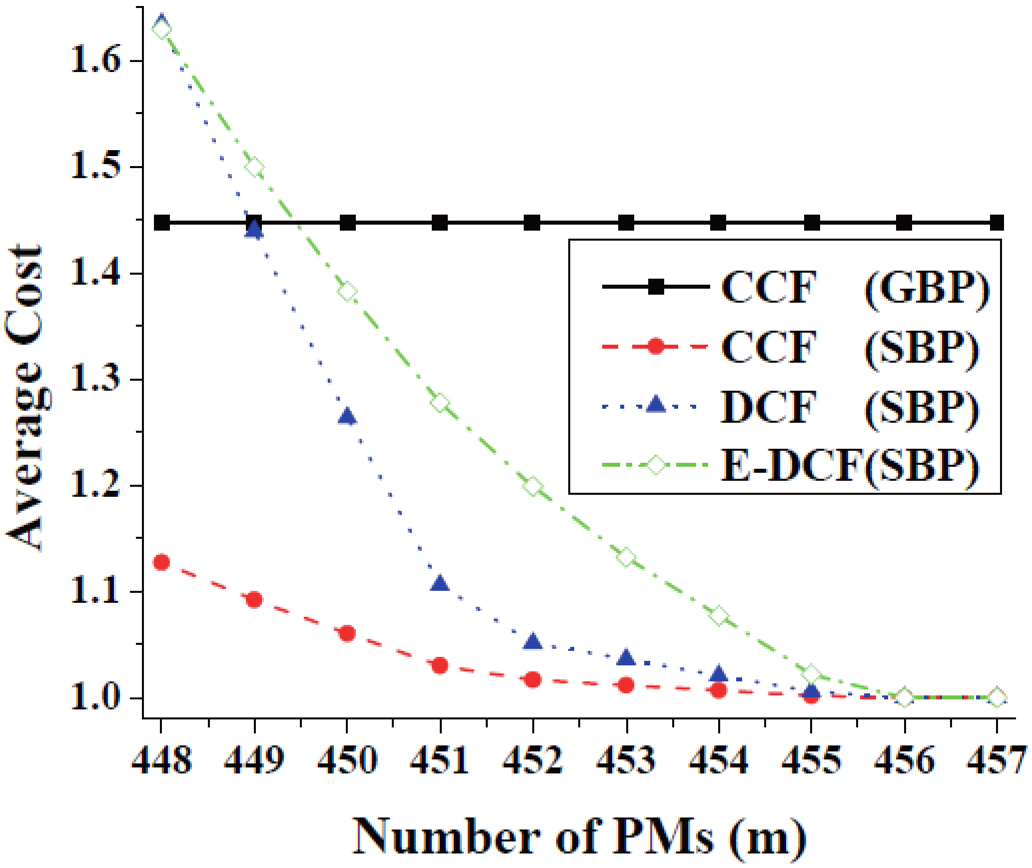


(b) DCF



(c) E-DCF

Impact of Number of PMs



Conclusion

- VM placement for network cost minimization.
- Homogeneous case
  - Optimal solutions for 3 cost functions
  - CCF, DCF, E-DCF
- Heterogeneous case
  - Approximation algorithm
  - 2-approximation ratio for CCF.

Thank You!

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